Gifted and Talented Education

Year 7 Mathematics

Polyominoes

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Polyominoes

Context

1. School details:
   Pennant Hills High School is a comprehensive coeducational high school located in the Hills School Network, Northern Sydney Region. The school has a population of approximately 1400 students with a significant number of non-English speaking background (NESB) students. Pennant Hills High School enjoys strong community support and has a reputation of high academic and sporting achievement.

2. School programs for gifted students:
   Each year, two Year 7 classes are formed to cater for potentially gifted and talented students using information gathered from feeder schools.

   Teachers at Pennant Hills High School have identified gifted and talented students across all year groups and are currently developing and implementing a range of initiatives to support their learning. These include experiences which extend learning within the classroom context as well as opportunities to participate in talented students programs, camps and competitions, public speaking experiences, leadership development conferences and camps, and music and drama performances. Opportunities for participation are provided by both the school and other community agencies.

3. Unit description:
   This unit was created for Year 7 (Stage 4) mathematics students who have been identified as having particular strength in the area of mathematics. Such students may not have been placed in the two selective classes but have been subsequently identified as gifted or talented by their current classroom teacher. Identification at this stage is through a combination of factors: samples of student work; performance in assessment tasks and contribution to class discussion.

   The unit involves the investigation of polyominoes and focuses on the manipulation of two-dimensional figures. It includes the concept of similar figures at a very elementary level. Students completing this unit will have the opportunity to engage with ideas loosely connected to the *Mathematics Years 7–10 syllabus* whilst pursuing an interesting inquiry into geometry.

4. Strategies for incorporating the unit into classroom practice may include:
   - pre-testing to establish the appropriate entry level for individual students
   - compacting of the curriculum for gifted students creating opportunities for this extension work
   - allowing gifted students to engage with the unit while the rest of the class is consolidating stage outcomes appropriate to individual knowledge and skills
   - using a contract approach (see Appendix 1) to structure the unit for individual student needs and abilities allowing for:
     - negotiated multiple entry and exit points at different levels
     - elements of choice

Objectives

Knowledge, Skills and Understanding

Students will develop knowledge, skills and understanding:

- through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection
- in spatial visualisation and geometric reasoning.

Values and Attitudes

Students will:

- show interest and enjoyment in inquiry and the pursuit of mathematical knowledge, skills and understanding
- develop and demonstrate perseverance in undertaking mathematical challenges.

Outcomes

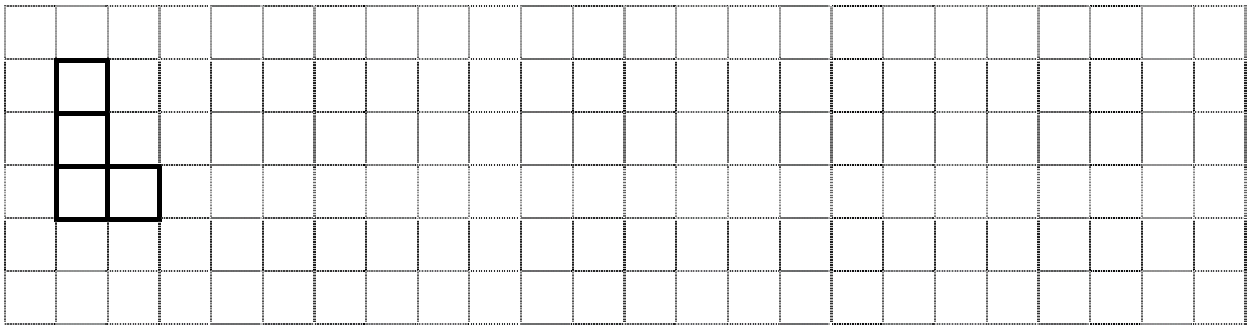
SGS3.2a **Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties**

SGS4.4 **Identifies congruent and similar two-dimensional figures stating the relevant conditions**

WMS4.2 **Analyses a mathematical or real-life situation, solving problems using technology where appropriate**

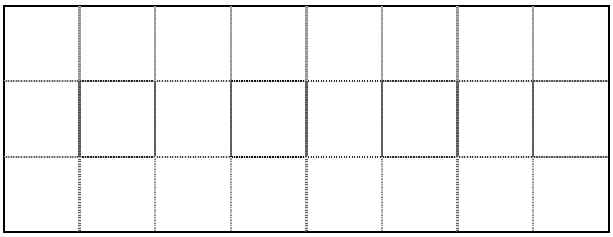
Learning Activities

1. Tetrominoes are shapes which are formed by four squares joined edge to edge in various combinations. One example is given here:



a) There are five possible shapes in a set of unique tetrominoes. Draw the other 4 tetrominoes. (Be careful – some may actually be the same tetromino in a different position.)

b) Divide the rectangle below using *only* tetrominoes.
   i) Complete the exercise using at least two *different* tetrominoes.
   ii) Complete the exercise using all the tetrominoes.



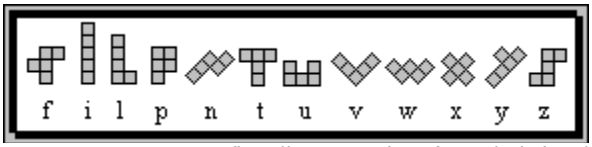
2. Play the game *Tetris* on a computer. A free version is available at www.neave.com/games/tetris/

3. The total area of the five tetrominoes is 20 squares but they cannot be joined together to form a single rectangle. Explain why this is so.

4. Pentominoes are shapes which are formed by five squares joined edge to edge in various combinations. There are twelve possible shapes in a set of unique pentominoes. Draw a set of unique pentominoes.

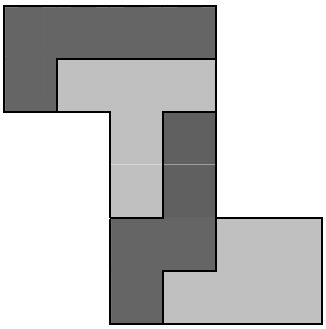
5. Use the pentominoes to build rectangles with given dimensions. A sample worksheet is provided in Appendix 2.
   c) Construct a 5×16 rectangle from the 12 pentominoes and the 5 tetrominoes.
   d) Build simultaneously a 3×5 rectangle and a 5×9 rectangle from the 12 pentominoes.
   e) Build simultaneously a 4×5 rectangle and 4×10 rectangle from the 12 pentominoes.

6. Duplication problems:
Each pentomino is referred to by the letter it resembles.



(http://www.puzzlecraft.com/solutions/pent/pentom/pentomin.html)

Four of the pieces can be used to make a Z twice as large as the original Z piece as illustrated below.



f) Duplicate the other pentominoes in a similar way.
g) There are two pentominoes that cannot be duplicated. Which ones are they?

7. Triplication problems:
Select any one of the pentominoes. Using only 9 of the remaining 11 pentominoes construct a shape, which is 3 times as large as the original.

8. Investigate the relationship between the areas of the original pentomino, its duplication and its triplication. What conclusions can be made? Investigate if this conclusion can be applied to shapes other than pentominoes.

9. Matching problems:
h) Two pentominoes can be fitted together to make a shape and then 2 *other* pentominoes can be fitted together to make the same shape. Show how I and L can be fitted together to make the same shape as W and N.
i) Find other pairs of pentominoes that can be matched in this way.

10. Dominoes are shapes that use two squares joined edge to edge. In a set of dominoes, the pairs of squares are then numbered using the digits 1 to 6 so that no domino is numbered in the same way.
j) How many dominoes are in a set?
k) Play the game of *Dominoes* and any of its variations.

11. The commercial game of *Tri-Ominoes* does not really use triominoes.
l) Explain why this is so.
m) Play the commercial game of *Tri-Ominoes*.
n) Search the Internet to find information about triominoes.
o) Use triominoes to develop a game.

12. Search the Internet for any other information on polyominoes. Find other problems involving polyominoes and present their solutions.

Assessment

Students undertaking this unit should record their work in a learning journal. In keeping this journal, students should be encouraged to record:

- solutions to the problems
- strategies used to solve the problems
- reflections on their thinking
- ideas for future exploration

This learning journal could then be used to assess student achievement of learning outcomes.

Students could also be asked to present their findings in a 10 minute presentation incorporating ICT.

The following is suggested as a means of assessing student learning:

Outstanding achievement will be evidenced by:

- multiple solutions to many problems
- excellent use of mathematical terminology in communicating concepts
- comprehensive explanations of the thinking employed throughout the problem solving process
- some ideas for future investigation

Good achievement will be evidenced by:

- multiple solutions to some problems
- good use of mathematical terminology in communicating concepts
- some explanation of the thinking employed throughout the problem solving process

Minimum achievement will be evidenced by:

- solutions to some problems
- some use of mathematical terminology in communicating concepts
- limited explanation of the thinking employed throughout the problem solving process

References

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Patjitnov, A. (c1986). *Tetris*, viewed 19 August 2005, www.neave.com/games/tetris/

Resources

Centre for Innovation in Mathematics Teaching, University of Exeter (2004) *Pentominoes – an introduction*, viewed 19 August 2005, http://www.cimt.plymouth.ac.uk/,

Gardner, M. (1959) *Mathematical puzzles and diversions*, New York, Pelican.

Jenicek, J. (2001) *The pentominoes page*, viewed 19 August 2005, http://www.puzzlecraft.com/solutions/pent/pentom/pentomin.html

Pairs of hexominoes in rectangles, viewed 19 August 2005, http://clarkjag.idx.com.au/PolyPages/index.htm?6pairs.htm

Putter, G. *Gerard's universal polyomino solver,* viewed 19 August 2005, http://www.xs4all.nl/~gp/PolyominoSolver/Polyomino.html

Wolfram Reasearch Inc. (1999) *Hexomino*, viewed 19 August 2005, http://mathworld.wolfram.com/Hexomino.html

Wolfram Reasearch Inc. (1999) *Polyomino*, viewed 19 August 2005, http://mathworld.wolfram.com/Polyomino.html

Tri-ominoes: the triangular domino game (1968) Croner Games, Maidstone, Vic.

Appendix 1: Sample contract

Name: …… Class: ……
Teacher: ……..
Unit: ……..

I agree to undertake the following activities and complete each task by the date indicated:

A. Non-negotiable tasks (numbers may vary):

1. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
2. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
3. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
4. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
5. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
6. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……

B. Negotiable tasks 9 numbers may vary):

7. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
8. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
9. ……
   Due date: …… Date completed: ……
   Student initials: …… Teacher initials: ……
10. ……
    Due date: …… Date completed: ……
    Student initials: …… Teacher initials: ……
11. ……
    Due date: …… Date completed: ……
    Student initials: …… Teacher initials: ……
12. ……
    Due date: …… Date completed: ……
    Student initials: …… Teacher initials: ……

I agree to complete all tasks as indicated on the above due dates:
Student signature: …… Date: ……..

I agree to provide assistance for ……..
Teacher signature: …… Date: ……

Appendix 2:

Pentominoes

Cut out each of the pentominoes below and then use the pieces to complete the challenges below.

Challenges:

1. Use the pieces to make a 5 × 3 rectangle.
   Draw at least 2 different solutions in your workbook.

2. Use the pieces to make a 5 × 4 rectangle.
   Draw at least 2 different solutions in your workbook.

3. Use the pieces to make a 5 × 5 square.
   Draw your solution in your workbook.

4. Mega-challenge:
   Fit the 12 pieces together to form rectangles of 6 × 10, 5 × 12, 4 × 15, 3 × 20.